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# Portfolio Optimization under Changing Conditions. Evidence from Europe.

**Abstract**

This paper proposes a procedure for dynamic optimization of an investment portfolio, consisting of stock market indices. SJC-copulas were used to assess statistical characteristics of assets. Copulas allow to measure interdependence between financial instruments, and to build further an efficient investment portfolio. As statistical characteristics of assets are changing with time, the structure of the portfolio is upgrading accordingly. The portfolio is then compared with two benchmarks in terms of return and risk. As a result, the proposed procedure provides better performance.

**Key words:** SJC-copulas, dynamic optimization of investment portfolio, interdependence among stock markets, Monte-Carlo simulation, CVaR.

**JEL classification:** C15, C61, C63, G11

# 1. Introduction

Europe can be considered one of the most advanced regions in terms of development of relations among the countries belonging to it. Some European countries have begun to develop economic cooperation in 1957 at the time of the creation of the European Economic Community. Since then, many measures have been designed to increase the number of members, and to facilitate the strengthening of trade relations and the formation of common goals. A number of institutions were founded to address a variety of problems of the union members. These institutions include the Council of the European Union, the European Commission, the Court of Justice of the European Union, European Court of Auditors, the European Central Bank and others. In addition, as part of further improvement of trade and tourism, in 1999 there was introduced the single European currency (only since 2002 in cash circulation).

Apparently, voluntary limitation of sovereignty in exchange for a membership in the European Union presents certain opportunities. On the other hand, as it became obvious in the light of the latest debt crisis in Europe, an excessive unity may lead to adverse consequences. This became obvious with the threat of default in Greece, when the problems of the Greek economy began to affect other EU countries.

Although concerns about external debt of particular European countries had started to rise in 2010, the main impact on stock markets occurred only in the second half of 2011 (EuroStoxx 50 index had dropped more than 30%). It may be the case that unity of European countries resulted in unity of their stock markets. That brings us to the main point of the study.

This paper examines interdependence of stock markets and proposes a procedure of building an efficient investment portfolio, taking into account the interdependence. Based on what was said above concerning Europe, it can be assumed that the interdependence of European stock markets was not maintained at a certain constant level. It is logical to assume that the certain stages of European Union development could contribute to changes in the level of correlation between stock markets of the members and, consequently, between their equity markets. The main events that affected the level of interdependence could be the introduction of the single currency, and joining the European Union by a particular country. This study first examines the fact of such changes, and then proposes a procedure for constructing an efficient portfolio in such a changing markets relationship.

To assess the level of interdependence among European stock markets special instruments are required. Based on the results of previous studies (more on that will be specified in the review of the literature) one of the acceptable options is the use of dynamic or conditional copula. The purpose of a copula is to build a joint distribution of two or more random variables. One of the advantages of this method is that it does not require a random variable to be elliptically distributed. This is quite important for studies of financial time series, which commonly do not have any known model for distribution. There are different types of copula and each of them assumes some particular form of a joint distribution. Parameters of copulas, fitted to particular time series, can be used to assess the level of interdependence. It is quite easy to find commonalities between measuring interdependence using copulas and simply calculating a correlation ratio. Still, in contrast to the latter, copulas do not require time series to be distributed normally.

In addition, some of the modifications to the copula method even allow the level of dependence to change in time, which is contrary to the correlation measurement preconditions. Using copulas it is possible to track changes in the level of dependence with each new observation. It is also quite important to emphasize, that the nature of dependence, measured by copulas is different from that of a linear correlation ratio. Copulas allow to track non-linear dependence, helping to conduct a more detailed study. In particular, dynamic copulas are used in this research to verify the hypothesis that the dependence of the stock markets is changing with time as a result of particular events – the introduction of a single currency, a particular country joining the European Union, or any kind of international agreements.

Dynamic copulas were also used to address the major problem considered in the study. That is building a stock indices portfolio, which is optimal in terms of risk/return ratio and takes into account changes in interdependence between assets. Copulas are used for Monte-Carlo simulation, which provides a proper estimate for the expected return, risk and dependence ratios of assets. On the basis of these estimates, it is possible to form an efficient portfolio frontier and chose a portfolio with the best return/risk ratio. This study in contrast to previous ones suggests dynamic portfolio optimization, that is active portfolio management using latest observations and interdependence estimates. The task is quite time consuming since Monte-Carlo simulations are required on each rebalancing of the portfolio. Nevertheless, it is worth checking whether such a procedure can offer some qualitative results of portfolio management. In this study the portfolio would be based on European stock indices. The performance of the portfolio would be compared with portfolios built by simpler methods.

The findings of the study can be helpful to portfolio managers with assets in different European countries. The proposed procedure probably would not be able to completely replace strategies and experience of professional managers, but still can be a useful auxiliary instrument for performance improvement. In addition, dynamic copulas can be useful to risk-managers, since they help to assess risk of a particular asset and a portfolio in general for different time periods. Results of the main hypothesis test may also be interesting to risk-managers. Based on the results it is possible to assume that changes in membership of European Union or Eurozone have to be accompanied by changes in models that explain dependence between stock markets of the countries involved.